TWISTED PHOTONS: WHAT IS THE ORBITAL ANGULAR MOMENTUM OF LIGHT?

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In this introductory article, we explain the concept of orbital angular momentum (OAM) of light, discussing its physical meaning and its relationship with the more familiar spin angular momentum of circularly polarized waves. We address both classical and quantum aspects, emphasizing the distinction between OAM eigenstates – helical waves – and the general case. Finally, we briefly touch upon the main optical techniques to generate these optical states and emerging applications in the field.

A*ngular momentum* is the rotational counterpart of (linear) momentum. In material systems, momentum can be interpreted as a measure of the "quantity of motion", where the contribution of each particle composing the system is given by the product of mass and velocity. Similarly, angular momentum along a given axis can be interpreted as a measure of the "quantity of rotational motion" around that axis. The contribution of each particle in this case is given by the product of its mass, angular velocity, and the square of its distance from the rotation axis. This particular measure of rotation is useful because it provides a *conservation law*, that is, the total angular momentum of an isolated system is always constant in time. Combining angular momenta along three orthogonal axes, one may construct the angular momentum vector, describing arbitrary rotational motions in space. The angular momentum vector $\mathbf{l}$ for a point-like material particle can be also directly computed by taking the cross product of the particle position $\mathbf{r}$ and the momentum $\mathbf{p}$, that is $\mathbf{l} = \mathbf{r} \times \mathbf{p}$. For an extended body, the total angular momentum $\mathbf{L}_{tot}$ is obtained by adding the contribution of all its small material parts. Interestingly, $\mathbf{L}_{tot}$ can then be decomposed into two terms, both referring to the whole body: (i) an "orbital" term associated with the motion of the body center-of-mass relative to the origin, given by $\mathbf{L}_o = \mathbf{R}_{cm} \times \mathbf{P}_{tot}$, where $\mathbf{P}_{tot}$ is the total momentum and $\mathbf{R}_{cm}$ the center-of-mass position vector; (ii) a "spin" term $\mathbf{L}_s$, arising from the possible rotational motion of the body about its own center. These two terms are for example respectively associated with the planets' orbital revolution around the Sun and spinning rotation around their axis.

Electromagnetic waves also "transport" conserved quantities, such as energy, momentum, and angular momentum, and can exchange them with matter during interaction. Being the waves distributed in space, all these quantities are defined in terms of spatial densities. The density of momentum can for example be computed as the cross product of the wave electric and magnetic fields times the vacuum permittivity, that is $\mathbf{p} = \varepsilon_0\mathbf{E} \times \mathbf{B}$ The density of angular momentum of the wave can then be given by $\mathbf{l} = \mathbf{r} \times \mathbf{p}$, just as for the case of matter. However, the total angular momentum of electromagnetic waves can then also be decomposed into the sum of two terms, commonly named *spin* and *orbital* angular momenta [1].

Spin angular momentum (SAM) is associated with the circular polarization of light. Orbital angular momentum (OAM) is generally associated with the waves' intensity and phase distribution in space. OAM eigenstates are helical waves, that is, waves characterized by a helical-shaped wavefront and having an optical vortex at their axis.

Spin angular momentum (SAM) is already familiar to most optics practitioners and is associated with the *circular polarization* of light. Left-handed and right-handed circular polarizations are respectively associated with a positive or negative angular momentum along the light propagation direction, which we will hereafter identify with the z axis (we adopt the "receiver-point-of-view" convention on the polarization left/right handedness). Quantitatively, the total SAM (z-component) of a circularly polarized wave is given by $L_{sz} = \pm U/\omega$ where U is the total wave energy, ω is its angular frequency ($\omega = 2\pi\nu$, where ν is the cyclic frequency), and the ± sign is fixed by the polarization handedness as mentioned above. Within a quantum description, each circularly polarized photon contributes with a spin of $\pm\hbar$ along its propagation direction, where $\hbar = h/2\pi$ is the reduced Planck constant (recall that each photon has an energy $U = h\nu = \hbar\omega$). Circular polarizations are the "*eigenstates*" of the photon spin, which means that these states have the property that, under a system rotation around the propagation axis, they remain unchanged except for a phase shift (or equivalently a time shift). In a quantum description, the eigenstates of any given quantity ("observable") correspond to states of the system that have a precisely known value of that quantity. Hence, circular polarizations are the only photon states in which their spin is precisely known. But then, what happens for linearly polarized light? Or more generally, for light having elliptical polarization? A linear or elliptical polarization can always be decomposed into the sum of two opposite circularly polarized components, with equal amplitudes in the case of linear polarization, unequal for elliptical ones. Classically, the spin of the wave will then just be the sum of the contributions of these two components. In the specific case of linear polarization, these two contributions are equal and opposite, and the total spin hence vanishes identically. In a quantum regime, the same superposition will instead define *probability amplitudes* for the photon spin. Hence, if the spin of a single photon is measured, it will randomly take one of the two nonzero values $\pm\hbar$, with probabilities given by the absolute-square of the corresponding amplitudes. If light is linearly polarized, each photon will then have 50% probability of having one of the values $\pm\hbar$. Therefore, although the mean value for the ●●●

photon spin still vanishes, as it does classically, no individual photon can have a vanishing spin.

Let us now consider the *orbital angular momentum* (OAM) of electromagnetic waves [1]. This quantity is generally associated with the waves' intensity and phase distribution in space, while it is essentially independent of the waves' polarization. A nonzero OAM is generally present in most electromagnetic waves. A trivial form of OAM for example occurs whenever a light beam has an optical axis that is not passing through the coordinates' origin. In this case, if $\mathbf{P}_{tot}$ is the total momentum transported by the beam and $\mathbf{R}$ is the position vector of a point on the beam axis (any point will do), there will be an OAM given by the cross product $\mathbf{L}_o = \mathbf{R} \times \mathbf{P}_{tot}$. This form of OAM, sometimes named "extrinsic" OAM, is fully analogous to the center-of-mass angular momentum of a material body. A more interesting form of OAM is identified when looking instead for OAM "eigenstates". As explained in Insert 1, *OAM eigenstates are helical waves,* that is, waves characterized by a helical-shaped wavefront and having an optical vortex at their axis (see fig. 1).

INSERT 1: OAM EIGENSTATES AND HELICAL WAVES

By definition, OAM eigenstates must have the property that, when subjected to a rotation around the propagation axis z (while holding the field orientation fixed), they remain unchanged except for a global phase shift. Hence, these particular waves must depend on the azimuthal angle φ around z (in the xy plane) only through a phase factor, *i.e.*, the (electric) field spatial dependence in cylindrical coordinates must assume the form $\mathbf{E}(r, z, \varphi, t) = \mathbf{E}_0(r, z, t)e^{im\varphi}$. Here, m is an integer constant characterizing the wave, which defines the phase shift $m\Delta\varphi$ acquired by the wave for arbitrary rotations by an angle $\Delta\varphi$. m must be integer because a full 2π rotation must restore the initial values of the field, otherwise the latter would not be a single-valued function. When including in the wave also the standard plane-wave phase factor, that is $\mathbf{E}(r, z, \varphi, t) = \mathbf{A}(r, z)e^{i(kz - \omega t)}e^{im\varphi}$, one obtains the following expression for the total optical phase: $\Phi = kz + m\varphi - \omega t$ (we ignore for simplicity possible radial-dependences of the phase). For a fixed time, this phase spatial dependence can be visually described by sketching the corresponding three-dimensional wavefronts (= loci of points having the same phase), which for $m \neq 0$ form helical-shaped surfaces (see fig. 1). More precisely, for $m \neq 0$, the wavefront forms $|m|$ intertwined helical surfaces, each with a longitudinal spatial period of $|m|\lambda$, where λ is the wavelength, and a helix handedness fixed by the sign of m. As a function of time, all these wavefronts will also move forward at the speed of light. Specific electromagnetic modes that have this "twisted" phase structure (but possibly different radial structures) include, for example, the Bessel and the Laguerre-Gauss modes. On the z axis the phase of these waves is undefined (for $m \neq 0$), hence the wave intensity must vanish identically, and one has an *optical vortex*. Besides the wavefronts, also the energy density of these waves flows around the propagation axis in a spiral fashion.

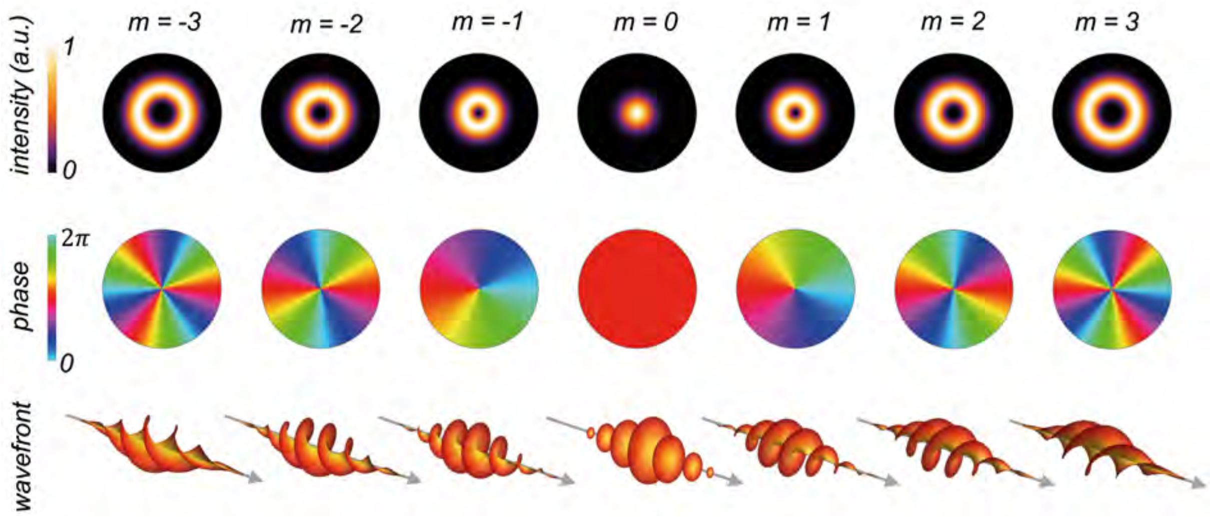

Figure 1. Structure of helical waves for different values of the integer m. The first row shows the typical doughnut-like intensity distributions in the xy transverse plane (in this example for the case of Laguerre-Gauss modes), with light vanishing at the center where the optical vortex is located (except for $m = 0$). The second row shows the phase distribution in a false-color scale. The third row shows the resulting multiple-helix wavefront (the gray arrow indicates the propagation axis z).

INSERT 2: TRANSVERSE OAM AND SPATIO-TEMPORAL VORTICES

In the main text we only discussed SAM and OAM oriented along the propagation direction (z axis), but "transverse" versions of these quantities, involving rotation around an axis that is orthogonal to propagation, may exist as well. For example, a transverse OAM can be defined in optical pulses that exhibit an $e^{im\varphi}$ phase factor in their field spatial dependence, where φ is now an azimuthal angle defined in the xz plane, which is combined with the usual propagation wave factor $e^{i(kz-\omega t)}$ (see fig. 2). As these pulses propagate, their z behavior is reflected into their temporal one, so they will exhibit spatio-temporal optical vortices (STOVs) [3].

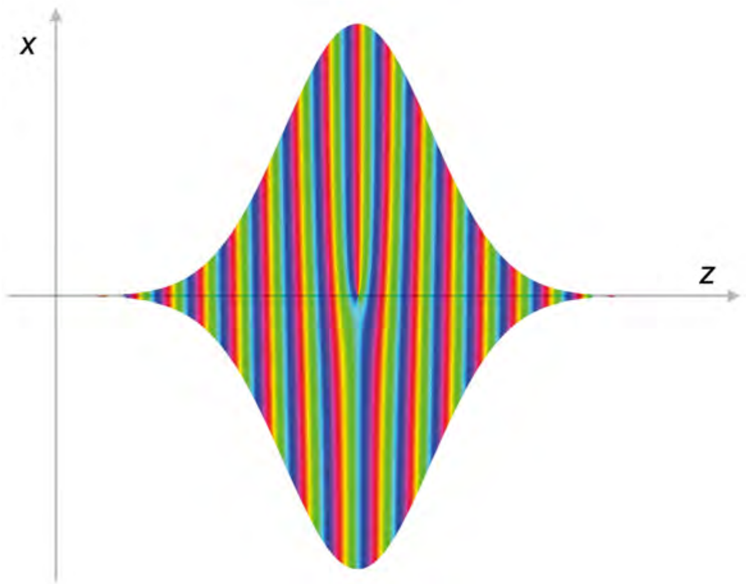

Figure 2. An optical pulse with "transverse" OAM. Shown are the pulse amplitude in the xz plane and the phase in false colors (see fig. 1 for the scale). The disclination defect in the phase pattern results from the vortex phase (for $m = 1$), which is superimposed to the ordinary plane wave one.

What is the angular momentum of these helical waves? It can be shown that the total OAM (z-component) is given by $L_{oz} = mU/\omega$, where m is the same integer that defines the helical phase structure (see Insert 1) [2]. In a quantum language this translates into an OAM of $m\hbar$ per photon, which comes in addition to the spin contribution. Notice that a single photon can in principle have an arbitrarily large angular momentum, since m is unbound. Just as in the case of spin and elliptical polarizations, a general wave is not necessarily an eigenstate of OAM, but it can always be decomposed into a superposition of such eigenstates, so that its OAM is given by the sum of their separate contributions. The corresponding photons will always have a specific value $m\hbar$ of OAM, when measured, which is however picked at random with probabilities proportional to the absolute-square of the corresponding superposition coefficients. Another important property of helical waves is that their total OAM is independent of the choice of coordinates' origin, unlike the "extrinsic" OAM mentioned above. Hence, despite being • • •

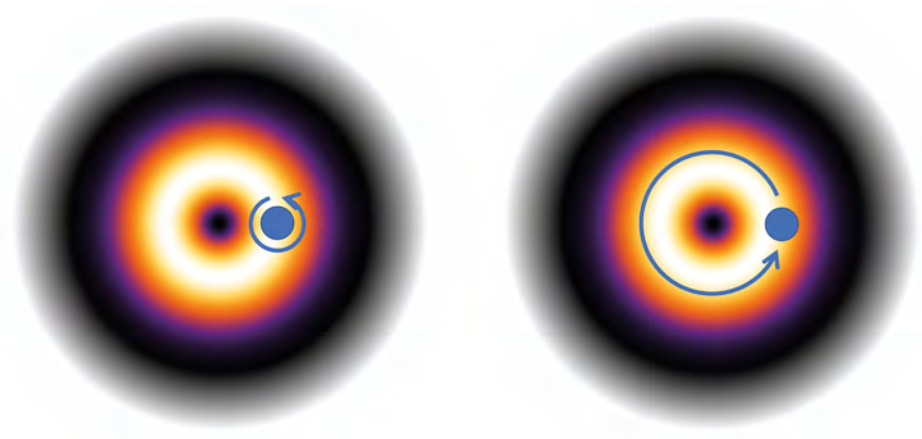

Figure 3. When a small absorbing particle localized off axis is placed in an optical beam that carries both SAM and OAM, it will be driven to spin around its axis by the SAM (left panel) and to orbit around the beam axis by the OAM (right panel).

"orbital", this OAM of helical waves is named *intrinsic* and, when compared with the case of material bodies, it behaves more as a second form of internal spin, rather than as the center-of-mass angular momentum.

The existence of angular momentum for circularly polarized light was first proved in the historical experiment by Richard A. Beth in 1936, in which a beam of light impinging on a suspended piece of birefringent material was shown to exert a tiny optical torque on it. The existence of the OAM form of angular momentum was similarly proved experimentally by using helical waves to set small objects in rotation and by controlling this mechanical rotation with the choice of the integer m [4]; an interesting difference is however found here, as explained in fig. 3. The measurement of OAM does not necessarily rely on its transfer to matter. Optical methods exist that, for example, exploit interference to detect the wave phase structure, or apply suitable complex optical transformations to convert different helical modes into separate light beams.

In conclusion, OAM is an intriguing property of light, associated with a nontrivial spatial structure. Over the past two decades, the field of optics has made significant progress in manipulating the spatio-temporal structure of light and exploring the related potential applications, often building on the prototypical example given by OAM. Today, this area of research constitutes a whole new subfield of optics known as "*structured light*" [5]. Helical modes and other complex forms of light are currently most easily generated by diffraction on computer-controlled holograms created by spatial light modulators (SLMs). Another convenient technology exploits the possibility of establishing a controlled *interaction between the spin and orbital forms* of angular momentum of light in suitable birefringent patterned media, such as liquid crystals or dielectric meta-materials. These innovative technologies, along with other recent advances, are also driving many studies on the possible applications of OAM and structured light, ranging from broad-band optical communication to enhanced optical microscopy, light-driven micro-motors, and high-dimensional quantum information processing and transmission. Who knows what other "twists" the future of optics may hold for us? ●

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